Neither Eliashberg equations nor the BCS one are predictive to the critical temperature, $T_c$, of a superconductor. A certain amount of phenomenological equations have been constructed with the aim of predicting the experimental values found for $T_c$. They preserve the form of the BCS-$T_c$ equation but introduce different combinations of the electron-phonon interaction parameter, $\lambda$, and the electron-electron repulsion parameter, $\mu^*$, that appear in the Eliashberg-$T_c$ equations. The agreement with experiment is, in general, rather poor. But since these are the only instrument that exists to predict $T_c$, they are widely used. A criterion that “the higher the $\lambda$, the higher the $T_c$” has emerged from those equations and the value of this parameter became the accepted criterion to discard or accept the electron-phonon interaction as a possible mechanism in HTSC. In this paper we analyze the theoretical foundations of this criterion and the validity of the criterion itself and compare to the results of Eliashberg-Migdal theory to conclude, first, that they contradict each other whenever we are dealing with HTSC and, second, that a low electron-phonon interaction parameter, $\lambda$, is not an argument solid enough to discard the e-ph interaction as a mechanism in HTSC.

I- Introduction

BCS theory [1] gives rise to an equation for the critical temperature of a superconductor, $T_c$, that depends on the attraction parameter, $V$, which cannot be accurately neither calculated nor measured. So, it cannot be used as a predictive equation, as it is well known. Eliashberg gap equations [2] allow the calculation of the critical temperature of a superconductor, $T_c$, ounce the Eliashberg function, $\alpha^2 F(\omega)$, and some other parameters are known or set. They are not, even then, predictive as well. This arises mainly because the electron-electron repulsion parameter, $\mu^*$, on which they depend, cannot be accurately neither measured nor calculated. When the Eliashberg function is unknown still a series of equations have been developed to predict $T_c$. In many cases, what is known is the electron-phonon interaction parameter, $\lambda$, which can be deduced from experiment or can be calculated as a normal state property. It is also related to the Eliashberg function as its first inverse moment. The magnitude of $\lambda$ has been widely used as an indicator of the magnitude of the critical temperature of a superconductor on the bases of a criterion derived from the just-mentioned set of empirical equations that preserve the exponential dependence of the BCS one and introduce different combinations of the electron-phonon interaction and the electron-electron repulsion parameters that enter in Eliashberg-Migdal theory (EMT). Others are approximations that derive from EMT and that contain, in addition, in certain cases,
parameters as the integral of the Eliashberg function, $A$, which is more complicated to establish without knowing the Eliashberg function itself. There is no precise theory on the adequate magnitude of the parameter, $A$, for a particular superconductor, but a kind of general agreement seems to exist that it should be kept, in any case, at reasonably low values [3]. There is a whole series of papers that deal with these approximate empirical equations [4]. They all support the criterion that “the higher the $\lambda$, the higher the $T_c$” and actually set a limit to $T_c$ by taking $\lambda$ to infinity to obtain the highest possible value of the exponential term. As a corollary, the impossibility emerges for a superconductor with a low value of $\lambda$ to be a candidate for a high $T_c$. That a high-$T_c$ material requires compulsorily a high value of $\lambda$, is the common “wisdom” nowadays. State-of-the-art calculations find systematically low values for $\lambda$ in HTSC [5] and the important question is whether or not this is an argument solid enough to discard the e-ph interaction as the mechanism responsible for the superconducting phase transition in these systems.

In this paper, we analyze the theoretical foundations of this criterion to show that, first, the correspondence between $\lambda$ and $T_c$ is rather poor even for low-$T_c$ materials; second, that it contradicts EMT, in some sense, whenever a high-temperature electron-phonon (e-ph) superconductor for which EMT holds is concerned (if it exists) and third, that to discard the e-ph mechanism on the basis of a low value of $\lambda$, is not theoretically well-founded. A low value of $\lambda$, as we show below, helps keeping the value of the parameter $A$ at a “reasonable” magnitude. Furthermore, the results from EMT seem to indicate that for high critical temperature superconductors, the parameter $\lambda$ could not contain anymore the crucial information on the factors that lead to the phase transition.

The rest of the paper is organized as follows. In the next section II, we deal with the $T_c$ equations to recall that none of them is really predictive and to introduce a few of the empirical equations developed to predict $T_c$ and to artificially set limits on it. In section III, we examine the criterion “the higher the $\lambda$, the higher the $T_c$” in some detail. Section IV is devoted to the functional derivative of $T_c$ with the Eliashberg function, $\alpha^2F(\omega)$. In this section, we draw most of the conclusions that sustain our arguments. In section V, we use the experimental and theoretical results to analyze the actual correspondence that exists between $\lambda$ and $T_c$. Our section VI answers to the question whether a low $\lambda$ value is such a bad result for an e-ph HTSC for which EMT holds (if it exists). In the final section VII, we summarize our arguments and draw our conclusions.

II- The $T_c$ equations and the limits to $T_c$.

BCS theory [1] gives rise to Eq. (1) for the critical temperature, $T_c$.

$$K_B T_c = 1.13 \hbar \omega_D e^{-1/N(0)V}.$$  

(1)

$K_B$ is the Boltzmann constant, $N(0)$ the density of electrons at the Fermi level, $\hbar$ is Planck’s constant, $\omega_D$ the Debye frequency and $V$ the attraction potential. This last parameter cannot be neither calculated nor determined experimentally. For that reason the BCS $T_c$-equation cannot actually be used to predict the critical temperature of a superconductor. Eliashberg gap equations [2] can be solved numerically to give an exact account of the thermodynamics of an electron-phonon (e-ph) conventional superconductors. We do not need to spell them down here. The interested reader is referred to the abundant existing literature (see the references quoted above, for
example). These equations can be linearized at \( T = T_c \) and can be solved once the needed data are given. The so-called Eliashberg function, \( \alpha^2 F(\omega) \), contains all the information on the existing phonons, the electron-phonon interaction and the included information on the conduction electrons in the material. One needs further to know accurately the electron-electron coulomb repulsion parameter, \( \mu^* \). Nevertheless, this parameter cannot be obtained neither theoretically nor experimentally with enough accuracy to be useful. So, in practice, the Eliashberg linear equations valid at \( T_c \) are used to fit a proper value for \( \mu^* \) to the experimental value of \( T_c \). The exact value of \( \mu^* \) depends also on the choice of the cut-off frequency that is necessary in order to end the infinite sum over the Matsubara frequencies. It is only after this fit is done that it is useful to proceed (using the same parameters) to solve the non-linear Eliashberg equations valid below \( T_c \) from which the thermodynamics of the corresponding superconductor can be calculated, resulting, in general, in very good agreement with experiment. Several codes have been produced with this purpose since long ago and they are widely known [6]. But, then, it is clear that Eliashberg equations cannot predict \( T_c \) even in the cases when the Eliashberg function, \( \alpha^2 F(\omega) \), is known. Actually, there is no reliable predictive equation for \( T_c \), nowadays.

This problem has been addressed replacing the unknown product \( N(0)V \) in the BCS \( T_c \)-equation (1) by different combinations of the electron-phonon interaction parameter, \( \lambda \), and the electron-electron coulomb repulsion parameter, \( \mu^* \), in an effort to reproduce the experimental results. The BCS-\( T_c \) equation can be transformed, for example, from Eq.(1) to

\[
K_B T_c = 1.13 \hbar \omega_D e^{\frac{1}{\lambda - \mu^*}}
\] (2)

where the denominator on the exponential \( N(0)V \) has been replaced by \( \lambda - \mu^* \). The idea is that the net attraction can be approximately described by both expressions. A BCS-like equation can be obtained from the linear Eliashberg gap equations by performing some replacements and assuming some equivalencies. The details can be found, for example, in ref. [3]. It results in Eq. (3)

\[
K_B T_c = 1.13 \hbar \omega_D e^{\frac{1+\lambda}{\lambda - \mu^*}}
\] (3)

which might actually be seen as the result of replacing \( N(0)V \rightarrow 1 + \frac{\lambda}{\lambda - \mu^*} \).

From Eq. (1), one can extract the criterion “the higher \( N(0)V \), the higher the \( T_c \)” which might be translated, on one hand, into “the higher the number of electrons at the Fermi energy, the higher the \( T_c \)” which is understandable as more cooper pairs can be formed from a higher number of electrons at the Fermi level and superconductivity is expected to be a more robust state in this case. On the other hand, the same condition can be interpreted as “the higher the pairing potential the higher the \( T_c \)” which is also understandable since a high pairing potential gives rise to a stronger bound in the pair state and it also translates in a more robust superconducting state. One might think that a stronger bound can be associated to a higher energy of the intermediate boson. This interpretation, as we shall see below, approaches BCS theory to the conclusions that derive directly from Eliashberg-Migdal theory (EMT).

The criterion that “the higher the \( \lambda \), the higher the \( T_c \)” follows from Eqs. (2) and (3), and several others of the kind which are derived from or related to Eliashberg gap equations in one way or another. There is a long series of papers [4] investigating
different versions of similar equations which, in essence, have all the exponential
dependence of the BCS Tc-equation but with coefficients and parameters that aim to
improve the agreement with experiment. It is important to stress that these equations are
related to but do not follow from EMT. They are not a direct consequence of it.

A very well known formula of this sort is the one first developed by Mc Millan [7] and later refined by Allen and Dynes [8]. It reads

\[ K_B T_c = \frac{\hbar \omega_{\ln}}{1.2} e^{-\left[ \frac{1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)} \right]} \]  

(4)

with

\[ \omega_{\ln} = \exp \left\{ \frac{2}{\lambda} \int_0^\infty \ln(\omega) \frac{\alpha^2 F(\omega)}{\omega} d\omega \right\} \]  

(4a)

This very popular formula deviates as much as 20% from the experimental value, in the
case of Pb. A point for discussion here is the adequate value for \( \mu^* \) to be used in this
formula that, on the other hand, requires the knowledge of the Eliashberg function,
\( \alpha^2 F(\omega) \), itself. To use Eq.(4), \( \mu^* \) is to be guessed. But, then, it is not clear whether the
same guess for \( \mu^* \) does not give a better approximation to Tc, when it is used to solve
directly the linear Eliashberg equations, a task which, in such conditions, is almost as
trivial nowadays as to use Eq. (4).

Limits to the value of Tc are found by taking \( \lambda \) to infinity in Eqs. (3) or (4) or in
similar equations. These equations give a finite, usually, different number for the
highest possible critical temperature. In other words, they set a definite limit to the
critical temperature of a superconductor. EMT sets actually no limit on the possible
values for a high temperature superconductor. We will come back to this point below.
These limits should obviously exist but might have to come from outside EMT as from
lattice instability or other arguments.

III- “The higher the \( \lambda \), the higher the Tc”.
The electron-phonon interaction parameter, \( \lambda \), is a property of the normal state that can
also be obtained from the Eliashberg function, \( \alpha^2 F(\omega) \), through the following formula,

\[ \lambda = 2 \int_0^\infty \frac{\alpha^2 F(\omega)}{\omega} d\omega \]  

(5)

\( \lambda \) is therefore the first inverse momentum of the Eliashberg function, \( \alpha^2 F(\omega) \). It is clear
that if a high value of lambda would be compulsory for a high value of Tc, then the
most important phonon frequencies would be the lower ones. In other words, to have a
high-Tc superconductor, its Eliashberg function should have most of its intensity at the
lowest possible frequencies since \( 1/\omega \) becomes a high weighting factor at low
frequencies (see Eq. (5)). We will show below that this conclusion is in contradiction
with EMT whenever a high temperature superconductor is concerned.

IV- The functional derivative of Tc with respect to \( \alpha^2 F(\omega) \), \( \delta Tc/\delta \alpha^2 F(\omega) \).
An important question to answer nowadays is whether it is possible for a
superconductor with a low electron-phonon interaction parameter, \( \lambda \), to have a high
critical temperature. We address this question under the assumption that the superconductor in question is appropriately described by the e-ph EMT.

Let us first recall some of the properties of the functional derivative of $T_c$ with respect to $\alpha^2 F(\omega)$, $\delta T_c / \delta \alpha^2 F(\omega)$, that will be useful to our arguments in this paper. It is important to understand the meaning of this function in detail. The answer to the question, how does the critical temperature change ($\Delta T_c$) when we induce a small change in the Eliashberg function ($\Delta \alpha^2 F(\omega)$) at some particular frequency, $\omega$, (through doping, for example), is given precisely by this function, as follows:

$$\Delta T_c = \int_0^\infty \frac{\delta T_c}{\delta \alpha^2 F(\omega)} \Delta (\alpha^2 F(\omega)) d\omega$$

(6)

We do not need to write explicitly the exact formula for this functional derivative here. It can be found in several places [9]. We only need here to know some of its characteristics. We merely show here how it looks like qualitatively (see Fig. 1). It rises linearly from zero and shows a maximum at a certain frequency which is characteristic to each superconductor. Then it slowly goes monotonically to zero as $\omega$ goes to infinity. But its most important property is that the just mentioned maximum is universal, this means that it is located at a frequency, $\omega_{opt}$,

$$\hbar \omega_{opt} = 7 K_B T_c$$

(7)

called the optimal frequency. The universal law that the optimal frequency, in units of the critical temperature, is equal to 7 for any superconductor for which EMT holds, says, in other words, that the $T_c$ is determined by the optimal frequency directly.

![The functional derivative of $T_c$ with the Eliashberg function. Its universal maximum is at $\hbar \omega / K_B T_c = 7$ for all superconductors for which EMT holds (see text).](image-url)
It is clear from Eq. (5) that if to get a high Tc, a high value of \( \lambda \) is required, the important part of the spectrum (\( \alpha^2 F(\omega) \)), lies at low frequencies. But to get a high Tc value, according to Eq.(7) which is the result that EMT actually gives, the optimal frequency should lie at higher values and therefore the high frequencies are the important ones. When we deal with superconductors that have a high Tc, the criterion “the higher the \( \lambda \), the higher the Tc” is therefore in contradiction with EMT.

An important question to answer nowadays, as we already wrote above, is whether it is possible for a superconductor with a low electron-phonon interaction parameter, \( \lambda \), to have a high critical temperature. Since, according to EMT, a high critical temperature is determined by the behavior at high frequencies, what happens at low frequencies is unimportant since it does not influence the value of Tc in an essential way. So, there is nothing that can be derived from EMT against an \( \alpha^2 F(\omega) \) with very low weight at low frequencies (with a low \( \lambda \)) and a high optimal frequency which would determine a high Tc. As an example, an Eliashberg function as the one sketched in Fig. 2 could represent an e-ph superconductor with a high Tc and a low \( \lambda \) under the condition that its optimal frequency lies at higher frequencies.

Fig.2- An hypothetical Eliashberg function that represents a superconductor for which the EMT holds with very little weight at low frequencies (low \( \lambda \)) and an optimal frequency lying at a very high frequency (say 70 meV) which would result in a Tc value of 10 meV, i.e. around 110K).

It is not possible to predict Tc nowadays but, nevertheless, the functional derivative \( \delta Tc/\delta \alpha^2 F(\omega) \) can be successfully used to answer more modest questions as, for example, whether a system is optimized or not. An analysis of this sort for Nb3Ge can be found in ref [10]. This work shows explicitly that what is important is the energy at which the optimal frequency lies. For low-Tc materials, \( \omega_{opt} \) lies at low frequencies and therefore the important part of the spectrum is also “in the picture captured” by \( \lambda \). This will not be the case for high-Tc materials. We will come back to this point below.
V- The experimental values for $T_c$ and $\lambda$: Do they really correlate?

If “the higher the $\lambda$, the higher the $T_c$” a certain correlation among the two values is expected. We saw from Eq.(5) that to get a high $\lambda$, the spectrum ($\alpha^2 F(\omega)$) should have some important weight at low frequencies. On the other hand, Eq. (7) reveals that the optimal frequency, $\omega_{opt}$, determines $T_c$. One expects therefore that both should then tend to agree better for low temperature superconductors where the optimal frequency, $\omega_{opt}$, lies at low frequencies. This also gives some theoretical foundation to the success of the idea that a correspondence exists between the magnitude of $\lambda$ and the one of $T_c$. What is really behind it is that, for low temperature superconductors, $\omega_{opt}$ lies at low frequencies, namely the ones that contribute the most to the value of $\lambda$. We examine that possibility in Table I. The data were taken from [4].

Table I contains four columns. The first from the left contains the value of the critical experimental temperature, $T_c$, in meV, and in ascending order and the second column shows the corresponding material. Notice that there are three insets for Nb with the same temperature. This is due to the fact that there are three different data for the corresponding value of $\lambda$. The third and fourth columns contain the material and the corresponding value of the electron-phonon interaction parameter, $\lambda$, again in ascending order from top to bottom. Would a perfect correspondence between $\lambda$ and $T_c$ existed, the names of the elements in column three and four would correspond to each other.

<table>
<thead>
<tr>
<th>$T_c$ [meV]</th>
<th>Material</th>
<th>Material</th>
<th>Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1017</td>
<td>Al</td>
<td>Al</td>
<td>0.43</td>
</tr>
<tr>
<td>0.2034</td>
<td>Tl</td>
<td>Ta</td>
<td>0.69</td>
</tr>
<tr>
<td>0.2931</td>
<td>In</td>
<td>Sn</td>
<td>0.72</td>
</tr>
<tr>
<td>0.3233</td>
<td>Sn</td>
<td>Tl</td>
<td>0.8</td>
</tr>
<tr>
<td>0.3612</td>
<td>Hg</td>
<td>V</td>
<td>0.8</td>
</tr>
<tr>
<td>0.3862</td>
<td>Ta</td>
<td>In</td>
<td>0.81</td>
</tr>
<tr>
<td>0.434</td>
<td>La</td>
<td>Mo</td>
<td>0.9</td>
</tr>
<tr>
<td>0.4621</td>
<td>V</td>
<td>La</td>
<td>0.98</td>
</tr>
<tr>
<td>0.5267</td>
<td>Bi</td>
<td>Nb(Rowell)</td>
<td>0.98</td>
</tr>
<tr>
<td>0.6198</td>
<td>Pb</td>
<td>Nb(Arnold)</td>
<td>1.01</td>
</tr>
<tr>
<td>0.7379</td>
<td>Ga</td>
<td>Nb(Butler)</td>
<td>1.22</td>
</tr>
<tr>
<td>0.7586</td>
<td>Mo</td>
<td>Pb</td>
<td>1.55</td>
</tr>
<tr>
<td>0.7931</td>
<td>Nb</td>
<td>Hg</td>
<td>1.62</td>
</tr>
<tr>
<td>0.7931</td>
<td>Nb</td>
<td>Ga</td>
<td>2.25</td>
</tr>
<tr>
<td>0.7931</td>
<td>Nb</td>
<td>Bi</td>
<td>2.45</td>
</tr>
</tbody>
</table>

Table I. The value for the experimental $T_c$ and for $\lambda$ for several elements.

It is clear from this Table that there is no general correspondence between the values of the electron-phonon interaction parameter, $\lambda$, and the value of the critical temperature, $T_c$. The correlation works well for Al, but extremely bad for Hg, for example. So the statement “the higher the $\lambda$, the higher the $T_c$” is not supported by the corresponding values presented here for several conventional superconductor elements. An exact correspondence does not exist even in the most favorable case.
The correspondence between $\lambda$ and $T_c$ is not much better for the A15 compounds. We compare in the same way as in Table I, the corresponding values in the next Table II. The same conclusion can be drawn in this case. $\lambda$ and $T_c$ are not, in general, related with each other as it should be expected if $\lambda$ were the proper parameter that determines the magnitude of $T_c$.

<table>
<thead>
<tr>
<th>$T_c$ [meV]</th>
<th>Material</th>
<th>Material</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4741</td>
<td>V3Si_1</td>
<td>V3Si_1</td>
<td>1</td>
</tr>
<tr>
<td>1.207</td>
<td>Nb3Al (2)</td>
<td>V3Si (Kihl.)</td>
<td>1</td>
</tr>
<tr>
<td>1.2931</td>
<td>V3Ga</td>
<td>V3Ga</td>
<td>1.14</td>
</tr>
<tr>
<td>1.4132</td>
<td>V3Si (Kihl.)</td>
<td>Nb3Al (2)</td>
<td>1.2</td>
</tr>
<tr>
<td>1.5603</td>
<td>Nb3Sn</td>
<td>Nb3Ge (2)</td>
<td>1.6</td>
</tr>
<tr>
<td>1.6121</td>
<td>Nb3Al (3)</td>
<td>Nb3Al (3)</td>
<td>1.7</td>
</tr>
<tr>
<td>1.724</td>
<td>Nb3Ge (2)</td>
<td>Nb3Sn</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table II. The A15 compounds [4]

This conclusion throws serious doubts on the use of the lambda parameter as an indicator of the magnitude of $T_c$. It seems that all the approximate equations that calculate $T_c$ based on the knowledge of $\lambda$ and $\mu^*$ alone or introduce extra sophisticated parameters as the $\omega_{\text{log}}$ in Mc Millan’s equation (4) could be misleading when they are applied to HTSC since they stress the behavior at low frequencies which is actually unimportant in this case according to EMT. Furthermore, these equations are not very precise even for low temperature conventional superconductors as we already mentioned (Pb). We also stress, referring to the Mc Millan’s equation (4) that if we know the Eliashberg function and have to guess the electron-electron repulsion parameter, with the same data and effort, we can solve directly the Eliashberg equations at $T=T_c$ and there is no reason a priori for the result not to agree better with experiment. In the worst case, there is at least a clearly set theory behind it.

The optimal frequency is calculated from the known experimental value of the critical temperature and Eq.(7) cannot be used to predict it.

What determines that an specific superconductor has a certain optimal frequency? This is an unsolved formidable problem that has not yet been addressed in the literature at least with enough detail as it is required. To solve it is actually equivalent to finding a predictive equation for $T_c$. Nevertheless, Eq.(7) is very useful to shed some light on whether or not the e-ph mechanism can be discarded solely on the basis of a small value of $\lambda$ as we shall see, in a model, below.

VI- Is a low value for $\lambda$ so bad for a HTSC?
As a final point in this paper, we explore the effect that a low $\lambda$ has on the value of an important parameter, namely, the integral of the Eliashberg function, $A$.

$$\int_0^\infty \alpha^2 F(\omega) d\omega \equiv A$$
Let us assume a delta-function model for the Eliashberg function. The highest $T_{c}$ will occur if all its weight is located at the optimal frequency according to Eq.(7):

$$\alpha^2 F(\omega) = A\delta(\omega - \omega_0) \quad \text{with} \quad \omega_0 = \omega_{\text{opt}}$$  \hspace{1cm} (9)

From Eqs.(5) and (9), we get

$$\lambda = \frac{2A}{\omega_{\text{opt}}}.$$  \hspace{1cm} (10)

This equation, in a certain way, is telling us, “the higher the $T_{c}$, the lower the $\lambda$” at least in the case when the parameter $A$ does not increase very much. This parameter should not be very high in any case [3]. It is not clear whether a too high value of the parameter $A$ is compatible with lattice stability.

In HTSC phonons of high frequency have been identified [11] For $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$, let us consider the phonon frequency at 50 meV. The amplitude of a model Eliashberg function for this material [12] appears to be maximal at this frequency. Would this phonon act as the optimal frequency, $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$ would have, according to Eq. (7) a $T_{c} \sim 7 \text{ meV} \sim 77 \text{ K}$. Since its experimental $T_{c}$ is around 30 K, we could conclude that, according to EMT, it is not an optimized system. We could say even more, since the Sr-content dependence of $T_{c}$ is fully known for this system and the highest $T_{c}$ found is quite lower than 77K, we can speculate that a higher $T_{c}$ could be obtained but not through the change of the Sr-content but with some different kind of impurities. This is not surprising if we take into consideration the way how the “on-earth temperature” (186K) new superconductor [13] has been obtained. Of course, there is no proof at all that the mechanism in the $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$ system is e-ph.

If we take 50 meV as the optimal frequency and believe that a high value of $\lambda$ is the proper way to get a high $T_{c}$, then we are forced to consider for $\lambda$ values of the order of 3 or so (for Nb: $T_{c}$=9 K, $A$=9 meV, and $\lambda$=1), then we get an embarrassing high $A = 75$ meV. But, fortunately, the result of a state-of-the-art calculation [14] is $\lambda$ of the order of 0.2, a value that brings back the parameter $A$ to a very reasonable magnitude, namely, about 5 meV, which is even lower than the one for Nb. In some conventional superconductors, as La, for example, “higher” values of $T_{c}$ are found together with lower values of $A$ [15]. So, from all these arguments, we can conclude that it might turn out that a low $\lambda$-value is, contrary to the accepted wisdom nowadays, even convenient for an e-ph HTSC for which the EMT holds, since it helps maintaining the parameter $A$ reasonable low.

VII- Conclusions

In the first part of this paper, we have analyzed the basis for the criterion “the higher the electron-phonon interaction parameter, $\lambda$, the higher the superconducting critical temperature, $T_{c}$”. This criterion is based on approximate phenomenological equations that are not a direct consequence of Eliashberg-Migdal theory (EMT) and they not only are not very precise even for the simple superconducting elements but also use values for the parameter $\mu^*$ that are guesses not well founded neither theoretically nor experimentally. We have then proceeded to show the results of EMT concerning the functional derivative of $T_{c}$ with respect to the Eliashberg function, $\alpha^2 F(\omega)$, $\delta T_{c}/\delta \alpha^2 F(\omega)$, to conclude that there is a universal law that relates $T_{c}$ directly to a particular phonon frequency, the optimal frequency, $\omega_{\text{opt}}$, that is specific to each
superconductor. So, to get a high critical temperature, contrary to the conclusions of the criterion based on the parameter $\lambda$, it is the high frequency part of the spectrum that is the important one. This comes directly from EMT and throws serious doubts on the validity of the conclusions that discard the electron-phonon mechanism on the basis of a low value of $\lambda$. Furthermore, we have shown that a low value of $\lambda$ helps to prevent an e-ph HTSC (if it exists) from having an unreasonable value of the integral of the Eliashberg function, $A$. It is not clear whether a too big value for the parameter $A$ could not be associated with the instability of the lattice. Summarizing, we argue, first, that the phenomenological equations based on the knowledge of $\lambda$ and $\mu^*$ alone as well as others requiring the knowledge of data that allow the solution of the Eliashberg equations themselves (as the Mc Millan one) should not be used in high-Tc superconductivity and, second, that a low value of the electron-phonon (e-ph) interaction parameter, $\lambda$, is not an argument solid enough to discard the e-ph mechanism in HTSC. We add a final argument. According to EMT, as the critical temperature of a superconductor increases (according to EMT) phonons of each time higher frequencies become involve in the dynamics of the phase transition and the lower frequencies do not matter so much because they do not play an essential role anymore, $\lambda$ ceases to capture enough details on the essential characteristics of the superconducting state. For that reason, we conclude that it is not clear whether $\lambda$ contains enough information so that it can be used as a proper criterion to accept or discard the e-ph mechanism in the case of HTSC,

References
[3] J. P. Carbotte, Rev. Mod. Phys. 62, 1027 (1990). This paper contains a review on some important part of the theoretical knowledge on superconductivity prior to the HTSC, studies the knowledge of HTSC at the time and compares with experiment at several places.
Estoy escribiendo una serie de trabajos (este es el primero) para tratar de introducir una nueva idea. La idea es que el parámetro de interacción electrón-fonón, \( \lambda \), muy usado en superconductividad convencional, podría dejar de tener significado cuando se trata de los nuevos superconductores con temperatura crítica alta. Los puntos importantes son:

1- Ese parámetro es el primer momento inverso de la función de Eliashberg. Esta función contiene los datos acerca de los fonones, la interacción electrón-fonón y sobre los electrones de la banda de conducción que son los que producen los pares de Cooper (una especie de molécula de dos electrones) responsable de la transición al estado superconductor.

2- La idea que reina aún es que una temperatura crítica elevada sólo se puede presentar en un superconductor que tenga un valor del parámetro \( \lambda \) elevado. Esto implica que la función de Eliashberg debe tener una amplitud grande a bajas frecuencias (la integral pesa la función de Eliashberg con \( 1/\omega \) –ver fórmula (5)- que tiende a infinito en cero, es decir, que las frecuencias bajas van a tener un peso muy grande en la integral). Este criterio deriva de ecuaciones fenomenológicas.

3- La Teoría de Eliashberg es la formulación en teoría de campo de la superconductividad. Bajo ciertas circunstancias, esta formulación da una descripción muy exacta de los superconductores convencionales. Uno de sus resultados importantes es la Ec. (7) que dice que la temperatura crítica está directamente determinada por una frecuencia especial, llamada la frecuencia óptima (ver texto para más detalles). Si se quiere obtener una temperatura crítica alta (según la teoría de Eliashberg) lo que importa es la parte de alta frecuencia de la función de Eliashberg ya que \( T_c \) grande implica que la frecuencia optima sea grande (ver Ec. (7)).

4- Si creemos en la teoría de Eliashberg, la física de la superconductividad debe buscarse en las \textbf{altas} frecuencias de la función de Eliashberg y \textbf{no en las bajas}, como lo requiere un valor grande del parámetro \( \lambda \). Por esta razón creo que los cálculos muy detallados y mut recientes de este parámetro, no pueden decir mucho sobre el origen de la superconductividad de alta temperatura crítica ya que “ese dato” no está ahí.

En esto baso mi idea de que \( \lambda \) podría perder la información importante (la de altas frecuencias) cuando el superconductor es de alta temperatura crítica y, en consecuencia, la formulación actual completa hacia el estudio de la interacción electrón-fonón en los nuevos materiales debe reestructurarse.