

Effective thermal conductivity and thermal diffusivity of two layer samples in photoacoustic experiments.

G. González de la Cruz, Yu. G. Gurevich., G. N. Logvinov, N. Muñoz Aguirre(*).
 Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional
 (CINVESTAV-IPN)

Apartado postal 14740, C.P. 07000 México D. F.

*Centro de Investigación en Ciencia Aplicada y Tecnología Avanzada del Instituto Politécnico Nacional
 (CICATA-IPN).

Legaria 694, Col. Irrigación, C.P. 11500, México D. F.

The effective thermal conductivity and thermal diffusivity are determined in one-dimensional bounded two-layer sample within the model of photoacoustic experiments. A new approach for calculating these thermal parameters is suggested. It is shown that effective parameters depend not only on the physical properties of separate layers, but on the manner of measuring these parameters and the points of measurements. Therefore, there exist several values of effective thermal conductivity and thermal diffusivity. The cases of low and high modulation frequencies are examined. The thermal property of interface is taken into account.

Keywords: Diffusivity, Photoacoustic, Photothermal, Isothermality, Superlattices.

Introduction

During the last years the use of layered materials has rapidly increased in a great number of applications such as the utilization of semiconductor heterostructures and especially the double heterostructures, including quantum wells, quantum wires and quantum dots in electronics. Superlattices, synthetic anisotropic samples, different polymers and biological objects, so on, are in an intent attention of the modern physics and technology also.

Among numerous physical properties of these materials, the thermal properties are one of the most important. Recently, for instance, the thermal properties of quantum wells, alternative layers of metal and semiconductors and superlattices^{1,2,3} have attracted the attention because of the promise of demonstrating greatly improvement of the performance of thermoelectric cooling and energy.

All objects enumerated above are essentially inhomogeneous, therefore, one important question arises about the determination of their effective heat parameters such as effective thermal conductivity and effective thermal diffusivity. Photoacoustic technique is widely used for experimental study of these parameters because its high sensitivity^{4,5}. The basic idea of the method is to place a sample in a cell containing a gas and a sensitive microphone. The sample is illuminated by means of a modulated laser beam and a nonstationary temperature is originated into the sample due to surface and bulk light absorption. The heat originated is transferred to the gas cell. Since the heating is modulated, the gas heating produces sound which is detected by the microphone. The detected signal contains information about thermal conductivity, thermal diffusivity, optical properties of mediums, so on. Generally, two different ways of gas heating are used: front surface illumination (closed photoacoustic cell) and rear surface illumination (open photoacoustic cell)^{6,7}. In Fig. 11 these two methods are shown on principle.

Two-layer structures are often examined as a simpler model of multilayer structures under theoretical research. The main problem is to get expressions for effective thermal parameters in terms of physical properties of the constituents materials which are in agreement with experimental measurements. Tominaga and Ito⁸, for example, studied the phase angle behavior as a function of the modulation frequency using the Rosencwaig-Gerso model⁹ for a two-layer system under rear-illumination. They showed that at high modulation frequencies the rear-illumination phase angle depends upon a critical frequency above which one of the layers becomes thermally thick. Another approach is suggested by A.M. Mansanares et al.¹⁰. They used the analogy between thermal and electrical resistance in heat transfer problems. The effective thermal diffusivity obtained by these authors depends on relative thickness fraction of materials, thermal diffusivities of each layer and the ratio of thermal conductivities of materials.

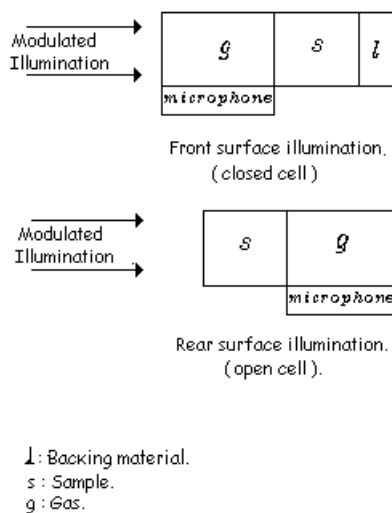


Figure 1. Photoacoustic cell configurations.

Recently, J.L. Lucio M. et al.^[11] have generalized the “resistance analogy” model. They have showed that formulas arisen from “thermal resistance analogy” model are valid only when both materials are thermally thin. In the case of thermally thick components, the formula for the effective thermal diffusivity was derived and they found that it depends, not only upon thermal parameters of constituents but on the chopping frequency too. The result of Tominaga and Ito^[8] was derived just in the case when the modulated frequency tends to infinity.

About the last mentioned works we have our opinion. The “electric and thermal analogy” is not adequate to describe a photoacoustic experimental situation. Potential difference only has physical sense in electrodynamics. In nonequilibrium thermodynamics, not only temperature difference has physical sense (the analogy with potential difference) but the temperature itself too; it can be measured at Kelvin absolute scale. Just the temperature or some response to this temperature, is measured in photothermal experiments, but not the temperature difference,

So, we suggest another approach for calculation of effective thermal conductivity and thermal diffusivity of two-layer samples. Every instrument measures temperature or temperature response in photoacoustic experiments only on surfaces of a sample, the volume is the “black box” for it. Therefore the main idea comes to an imaginary replacement of the real two-layer structure by an effective one-layer sample. The demand of equality of boundary conditions on the left or right surfaces are necessary. Thermal parameters of this effective sample are the effective parameters of the real two-layer structure.

This method was briefly described in [12] and results of paper [8] have been reproduced without conditions about constituents thermal thickness and without using “electric-heat analogies”.

It is important to emphasize that equalization of temperatures on either left or right sides of real two-layer system and effective one-layer sample (but not temperature difference) can lead to different expressions of effective parameters. It means that the resultant effective thermal conductivity and thermal diffusivity, in general, depend on how we influence the physical system and the point of measurement. With respect to photoacoustic experiments it means that we can measure different values of effective thermal conductivity and effective thermal diffusivity depending on which illumination method is utilized.

Another aspect that we consider in this paper is the influence of the interface between layers on the effective parameters. This problem has not been discussed in detail in investigation thermal parameters of two-layer systems under photoacoustic experiments. At the same time many experimental investigations (see, for example [13-16]) and theoretical study [17] points out existence of interface thermal conductance. It is clear that the effective thermal parameters must depend on this interface thermal property too, besides other factors mentioned already. Therefore, the

thermal conductivity h can be measured by photoacoustic technique.

The aim of this paper is to develop the stated conception for the case of arbitrary two-layer structures and arbitrary values of modulation frequencies. For simplicity, we examine only optically opaque samples.

a: Temperature distributions in two-layer structure and effective one-layer specimens

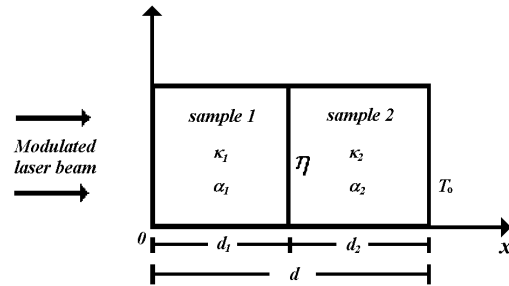


Figure 2. Two-layer structure.

Let us assume that energy of the modulated laser beam is converted into heat completely onto the surface $x=0$ of the two-layer sample which has a parallelepiped form (see Fig. 2). Each layer is homogeneous and isotropic and the cross section at any plane perpendicular to the axis OX is equal to unity. The opposite surface $x=d$ is in contact with a thermostat at a temperature T_0 ; the lateral sides are adiabatically insulated. Thus the problem is one-dimensional and the temperature distributions in two-layer structure can be obtained from the following heat diffusion equations:

$$\frac{\nabla^2 T_{1,2}(x,t)}{\nabla x^2} = \frac{1}{a_{1,2}} \frac{\nabla T_{1,2}(x,t)}{\nabla t}, \tag{1}$$

where $T_{1,2}(x,t)$ are temperatures, $a_{1,2} = \mathbf{k}_{1,2} / (\mathbf{r}c)_{1,2}$ are thermal diffusivities, $\mathbf{k}_{1,2}$ are thermal conductivities, $\mathbf{r}_{1,2}$ are densities and $c_{1,2}$ are specific heats in the first and the second layer, respectively.

We consider the intensity of the incident light weak enough and neglect, for this reason, the dependence of all kinetic coefficients on local temperature, so the equations (1) are linear.

In the steady state the equations (1) must be added only by the boundary conditions because of the harmonic temporal perturbations. Since we have assumed the illumination absorption happens at the surface exclusively, then, the heat boundary condition represents the surface heat source on the surface $x=0$. We also consider that the sample bulk thermal conductivity is much greater than the surrounding one, therefore the thermal flux flows into the sample completely.

In this problem we take into account the thermal properties of interfacial region between layers. Really it has some thickness and some thermal conductivity different

from the thermal conductivities in the principal volumes of the layers. In the model of surface interface its thickness tends to zero and the heat property of this region is described by the some surface thermal conductivity \mathbf{h} The temperature gap must takes place at the interface plane $x=d_1$ when the thermal conductivity \mathbf{h} has the finite magnitude^[17].In this case within the interface region the thermal flux is defined by the following temperature difference $T_2(x=d_1+0) - T_1(x=d_1-0)$.Simultaneously the condition of continuity of thermal fluxes must takes place at the plane $x=d_1$.

Similar to interface , the thermal flux on the surface $x=d$ must be determined by some surface thermal conductivity \mathbf{h} and by the temperature difference $T_0 - T(x=d-0)$, where T_0 is the ambient temperature. However for simplicity we consider the surface thermal conductivity \mathbf{h} is infinite ($\mathbf{h} \rightarrow \infty$) so the boundary condition on the side $x=d$ is reduced to equality of the sample and ambient temperatures (isothermal contact).

Thus the boundary conditions to equations (1) can be written in the following form:

$$-\mathbf{k}_1 \frac{\partial T_1(x,t)}{\partial x} \Big|_{x=0} = Q_0 + \Delta Q e^{i\omega t} \Big|_{x=0}, \quad (2.1)$$

$$\mathbf{k}_1 \frac{\partial T_1(x,t)}{\partial x} \Big|_{x=d_1} = \mathbf{k}_2 \frac{\partial T_2(x,t)}{\partial x} \Big|_{x=d_1}, \quad (2.2)$$

$$-\mathbf{k}_1 \frac{\partial T_1(x,t)}{\partial x} \Big|_{x=d_1} = \mathbf{h}(T_1(x,t) - T_2(x,t)) \Big|_{x=d_1}, \quad (2.3)$$

$$T_2(x,t) \Big|_{x=d} = T_0. \quad (2.4)$$

Here Q_0 is the average over time the high-frequency component of the total heat flux at the surface $x=0$, $\Delta Q e^{i\omega t}$ is the modulation component of it, ω is the modulation frequency. The static component Q_0 determines dc temperature distribution, while the dynamic part $\Delta Q e^{i\omega t}$ represents sinusoidal surface heat source.

The steady state solutions of equations (1) with boundary conditions (2) are as follows:

$$\begin{aligned} 0 \leq x \leq d_1, \\ T_1(x,t) = A_1 + B_1 x + \\ + \{U_1 \cosh[\mathbf{s}_1(x-d_1)] - U_2 \sinh[\mathbf{s}_1(x-d_1)]\} e^{i\omega t}, \quad (3) \end{aligned}$$

$$\begin{aligned} d_1 \leq x \leq d, \\ T_2(x,t) = A_2 + B_2(x-d_1) + \\ + \left\{ \frac{2 \left(\frac{\Delta Q}{\mathbf{k}_2 \mathbf{s}_2} \right) \sinh[\mathbf{s}_2(d-x)]}{D} \right\} e^{i\omega t} \end{aligned}, \quad (4)$$

where;

$$\begin{aligned} U_1 &= \frac{2 \left(\frac{\Delta Q}{\mathbf{k}_1 \mathbf{s}_1} \right) \left[\frac{\mathbf{k}_1 \mathbf{s}_1}{\mathbf{k}_2 \mathbf{s}_2} \sinh(\mathbf{s}_2 d_2) + \frac{\mathbf{k}_1 \mathbf{s}_1}{\mathbf{h}} \cosh(\mathbf{s}_2 d_2) \right]}{D}, \\ U_2 &= \frac{2 \left(\frac{\Delta Q}{\mathbf{k}_1 \mathbf{s}_1} \right) \cosh(\mathbf{s}_2 d_2)}{D}, \\ A_1 &= T_0 + \frac{Q_0}{\mathbf{h}} + Q_0 \left(\frac{d_1}{\mathbf{k}_1} + \frac{d_2}{\mathbf{k}_2} \right), \quad (5) \end{aligned}$$

$$A_2 = T_0 + \frac{Q_0}{\mathbf{k}_2} d_2,$$

$$B_1 = -\frac{Q_0}{\mathbf{k}_1},$$

$$B_2 = -\frac{Q_0}{\mathbf{k}_2},$$

$$\mathbf{s}_{1,2} = (1+i)q_{1,2},$$

$$q_{1,2} = \sqrt{\frac{\omega}{2\mathbf{a}_{1,2}}},$$

$$\begin{aligned} D &= \left(1 - \frac{\mathbf{k}_1 \mathbf{s}_1}{\mathbf{k}_2 \mathbf{s}_2} \right) \cosh(\mathbf{s}_1 d_1 - \mathbf{s}_2 d_2) \\ &+ \left(1 + \frac{\mathbf{k}_1 \mathbf{s}_1}{\mathbf{k}_2 \mathbf{s}_2} \right) \cosh(\mathbf{s}_1 d_1 + \mathbf{s}_2 d_2) \\ &+ \left(\frac{\mathbf{k}_1 \mathbf{s}_1}{\mathbf{h}} \right) \left[\sinh(\mathbf{s}_1 d_1 - \mathbf{s}_2 d_2) + \sinh(\mathbf{s}_1 d_1 + \mathbf{s}_2 d_2) \right] \end{aligned}$$

It is easy to see that the temperature of each layer depends on thermal bulk parameters of both layers and on the interface thermal conductivity too. It is obviously that the photoacoustic signal carries an information about some integral values of the thermal conductivity and the thermal diffusivity which are constructed from all parameters of both layers. What is this construction is the main question of the examined problem.

Let us now image some one-layer sample having the same geometric form as two-layer system with some thermal conductivity \mathbf{k} thermal diffusivity \mathbf{a} and the thickness $d=d_1+d_2$ (see Fig. 3).

Let us demand also the following boundary conditions:

$$-\mathbf{k} \frac{\partial T(x,t)}{\partial x} \Big|_{x=0} = Q_0 + \Delta Q e^{i\omega t} \Big|_{x=0}, \quad (6)$$

$$T(x,t) \Big|_{x=d} = T_0,$$

where $T(x,t)$ is the temperature distribution in the one layer sample, satisfying the thermal diffusion equation:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\mathbf{a}} \frac{\partial T(x,t)}{\partial t}. \quad (7)$$

It is easy to obtain, that

$$T(x, t) = T_0 + \frac{Q_0}{k} (d-x) + \left\{ \left(\frac{\Delta Q}{ks} \right) \frac{\sinh[s(d-x)]}{\cosh[sd]} \right\} e^{im} \quad (8)$$

where $s = (1+i)q$ and $q = (\omega/2a)^{1/2}$.

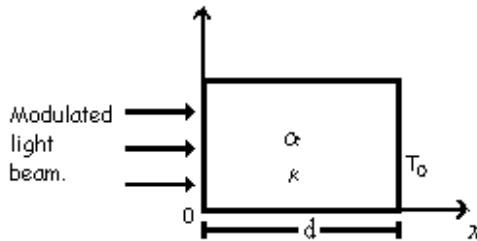


Figure 3. One-layer structure.

Let us call the values k_f and a_f the effective thermal conductivity and effective thermal diffusivity measured by front surface illumination method (FSIM) if they are obtained from the follow equation:

$$T(x=0, t) = T_1(x=0, t). \quad (9)$$

Similar one could define the effective thermal conductivity k_R and the effective thermal diffusivity α_R under the measurement by the rear surface illumination method (RISM), i.e. from the side $x=d$ and there is not problem in the case when surface thermal conductivity has a finite magnitude ($h \neq \infty$). If contacts are isothermal we can not compare temperatures $T_2(x)$ and $T(x)$ on the surface $x=d$ because they are equal here to the ambient temperature [see boundary conditions (2,4) and (6)]. So the procedure of obtaining thermal parameters by RISM must be concern the points closely spaced to $x=d$. The equality

$$T(x=d-\epsilon, t) = T_2(x=d-\epsilon, t) \quad (10)$$

is the equation to calculate k_R and a_R by RISM, where ϵ is arbitrary small value.

We postulate that the values of all this effective parameters are adequate to the experimental situation.

b: The effective thermal conductivity under static thermal fluxes.

The particular case of static thermal processes corresponds to the condition $\Delta Q=0$ [see Eqs. (2)]. The real temperature distributions in the layers are:

$$T_1(x) = T_0 + Q_0 \left(\frac{d_2}{k_2} + \frac{1}{h} \right) + \frac{Q_0}{k_1} (d_1 - x), \quad 0 \leq x \leq d_1. \quad (11)$$

$$T_2(x) = T_0 + Q_0 \frac{d_2}{k_2} + \frac{Q_0}{k_2} (d_1 - x), \quad d_1 \leq x \leq d. \quad (12)$$

Let us pay the attention to the fact that the temperature distribution undergoes to gap at the interface plane under arbitrary values of h except $h = \infty$ (isothermal contact),

$T_1(x=d_1) \neq T_2(x=d_1)$. Schematically the temperatures $T_1(x)$ and $T_2(x)$ are shown in Fig 4.

The effective static temperature is given by the follow expression (see Eq. (8)):

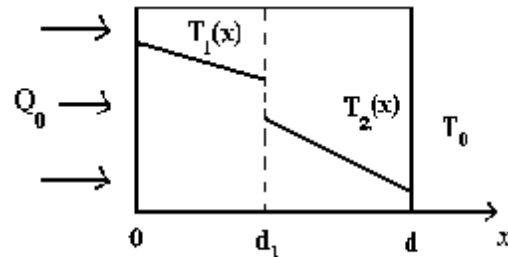


Figure 4. True temperatures $T_{1,2}(x)$ in the first and second layer.

$$T(x) = T_0 + \frac{Q_0}{k} (d-x), \quad 0 \leq x \leq d. \quad (13)$$

The behavior of this temperature essentially depends on the method of measurement. If FSIM is applied the $T(x)$ -plot is pointed in Figure 5, and the equality (9) reduces to the expression

$$k = \frac{d_1 + d_2}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{1}{h}}, \quad (14)$$

which coincides with the well known expressions [10], [19], when $h \rightarrow \infty$

The equality (10) (RISM) gives the another value of the effective thermal conductivity:

$$k = k_2. \quad (15)$$

In this case the function $T(x)$ is presented in Fig. 6. It coincides with the function $T_2(x)$ on the interval $d_1 \leq x \leq d$, and differs from the temperature $T_1(x)$ in the first layer.

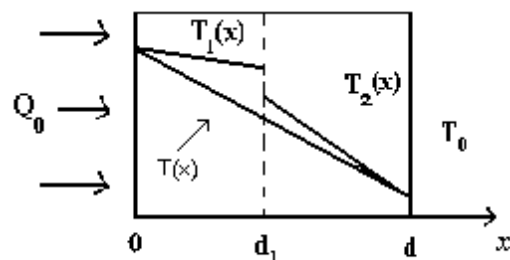


Figure 5. Effective temperature $T(x)$ under FSIM.

The explanation about the last result can be find in the following way. The second layer “does not fill” the first layer heat parameters when the static flux boundary conditions are given on the surface $x=0$ (boundary conditions of the second kind). The thermal conductivity k_2 , the ambient temperature T_0 , the heat flux Q_0 and the sample

thickness d determine temperature behavior $T_2(x)$, therefore RSIM comes to result (15).

On the contrary, temperature distribution $T_1(x)$ is determined by all heat and geometric parameters of two-layer system and by the magnitude Q_0 . So, the effective thermal conductivity measured by FSIM includes itself all this parameters.

It should be mentioned that the different values of the effective thermal conductivities are received only in the situations when just the flux boundary conditions are given on the surface $x=0$. This values coincide under measurements from the both sample sides if the boundary conditions are of the first or third kind [20].

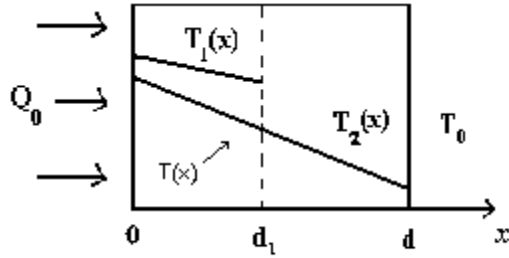


Figure 6. Effective temperature $T(x)$ under R.SIM.

c: The effective thermal conductivity and thermal diffusivity under dynamic thermal fields.

The situation becomes more complicated when the thermal field depends on time and space. In this case the temperature has a complex value and the equalities (9), (10) must be considered as the equalities of a real and image parts separately. This leads to the system of two equations for determination of the two unknown effective values k and a . After some mathematical transformations the expressions (9) and (10) reduce to the follow two equations systems:

$$\frac{\sin(qd)\cos(qd)}{\sinh(qd)\cosh(qd)} = \frac{D_R S_I - D_I S_R}{D_I S_I + D_R S_R}, \quad (16)$$

$$k = k_1 \frac{q_1}{q} \left[\frac{(D_R)^2 + (D_I)^2}{D_I S_I + D_R S_R} \right] \frac{\sinh(qd)\cosh(qd)}{\cos^2(qd) + \sinh^2(qd)}, \quad (17)$$

and

$$\tan(qd)\tanh(qd) = \frac{D_I}{D_R}, \quad (18)$$

$$k = \frac{k_2}{2} \frac{D_R}{\cos(qd)\cosh(qd)}, \quad (19)$$

where:

$$q = \sqrt{\frac{w}{2a}},$$

$$\begin{aligned} D_R &= b_1 + \frac{k_1 q_1}{k_2 q_2} b_3 + \frac{k_1 q_1}{h} (b_5 - b_6), \\ D_I &= b_2 + \frac{k_1 q_1}{k_2 q_2} b_4 + \frac{k_1 q_1}{h} (b_5 + b_6), \\ S_R &= \frac{k_1 q_1}{k_2 q_2} b_7 + \frac{k_1 q_1}{h} b_1 + b_5, \\ S_I &= \frac{k_1 q_1}{k_2 q_2} b_8 + \frac{k_1 q_1}{h} b_2 + b_6, \\ b_1 &= \cos I_2 \cosh I_2 + \cos I_1 \cosh I_1, \\ b_2 &= \sin I_2 \sinh I_2 + \sin I_1 \sinh I_1, \\ b_3 &= \cos I_2 \cosh I_2 - \cos I_1 \cosh I_1, \\ b_4 &= \sin I_2 \sinh I_2 - \sin I_1 \sinh I_1, \\ b_5 &= \cos I_2 \sinh I_2 + \cos I_1 \sinh I_1, \\ b_6 &= \sin I_2 \cosh I_2 + \sin I_1 \cosh I_1, \\ b_7 &= \cos I_2 \sinh I_2 - \cos I_1 \sinh I_1, \\ b_8 &= \sin I_2 \cosh I_2 - \sin I_1 \cosh I_1, \\ I_1 &= q_1 d_1 - q_2 d_2, \\ I_2 &= q_1 d_1 + q_2 d_2, \end{aligned} \quad (20)$$

$$q_1 = \sqrt{\frac{w}{2a_1}},$$

$$q_2 = \sqrt{\frac{w}{2a_2}}.$$

As it is easy to see, in general case, the effective thermal conductivity κ and effective thermal diffusivity α depends on the thermal conductivities, thermal diffusivities and thicknesses of each layer. Besides, they depends on the interface thermal conductivity and the chopper frequency. The systems of equation (16), (17) and (18), (19) are transcendental and can be solved exactly only by numerical methods. The analytical calculations are possible only in some special cases.

d: Special cases.

Let us discuss two groups of special cases corresponding to low and high modulation frequencies. Since the frequency $2a/d_i^2$ is characteristic frequency of i -th layer under propagation thermal flux in it, then the criterions of low and high frequencies are the following relations:

$$w \ll \frac{2a_i}{d_i^2} \quad (21)$$

and

$$w \gg \frac{2a_i}{d_i^2}. \quad (22)$$

It is clear that one and the same modulation frequency w can be different by its magnitude with respect to the different layers of the two-layer structure.

The inequalities (21), (22) can be rewritten in the another form

$$q_i d_i \ll 1 \tag{23}$$

and

$$q_i d_i \gg 1. \tag{24}$$

where $q_i^{-1} = \sqrt{\frac{2a}{w}}$ is the thermal diffusion length of i -th layer. The criterions (23), (24) determine thermally thin and thermally thick layer respectively.

I. Thin layers.

1. Let us suggest that both layers are thin: ($q_1 d_1, q_2 d_2 \ll 1$) and the additional condition $h^3 k_i / d_i$ takes place. Then the front surface illumination measurements (FSIM) [Eq (9)], lead to the following results:

$$k_F = \frac{d_1 + d_2}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{1}{h}}, \tag{25}$$

$$a_F = \frac{(d_1 + d_2)^3}{k_F \left(\frac{d_1^3}{a_1 k_1} + \frac{d_2^3}{a_2 k_2} \right) + 3 \frac{k_1 d_1 d_2 (d_1 + d_2)}{a_1 k_2} \left(1 + \frac{k_2}{h d_2} \right)}. \tag{26}$$

If the rear surface illumination measurements (RSIM) are used then [see Eq. (10)],

$$k_R = k_2. \tag{27}$$

$$a_R = \frac{d^2}{\frac{d_1^2}{a_1} + \frac{d_2^2}{a_2} + 2 \frac{k_1}{k_2} \frac{d_1 d_2}{a_1} \left(1 + \frac{k_2}{h d_2} \right)}. \tag{28}$$

The effective value of the thermal conductivity does not depend on the thermal diffusivities and is determined by the same expressions as in the static case. The effective thermal diffusivity value depends on the all heat parameters of the two-layer system such as the thermal conductivities k_1, k_2 ; thermal diffusivities a_1, a_2 ; layer geometric sizes and the interface thermal conductivity h

Let us note that the criterions of isothermality of the contact between layers are different depending on what heat effective parameters is considered. The interface contact is more heat transparent in the research of the effective thermal conductivity when

$$h \gg \frac{k_1 k_2}{k_2 d_1 + k_1 d_2}, \tag{29}$$

i.e. this criterion includes itself the interface thermal conductivity, the thermal conductivities of both layers and its thickness.

In the case of investigation the effective thermal diffusivity we must use the another criterion of interface isothermality:

$$h \gg \frac{k_2}{d_2}. \tag{30}$$

It is determined by the correlation between the interface thermal conductivity, the heat conductivity of the second layer and its thickness.

Let us note that the effective thermal parameters does not depend on the modulation frequency w for thermally thin films as under FSIM so under RSIM.

2. It is interesting to examine separately the influence of interface on the effective parameters values. Let us assume that the two-layer system consists of two identical layers ($d_1 = d_2, k_1 = k_2, a_1 = a_2$) divided by some interface having the finite surface thermal conductivity. The effective thermal parameter measurements by the method of closed or open cells reduce to the different values even in this elementary case. Really, under front illumination we have:

$$k_F = k_1 \frac{1}{1 + \frac{k_1}{2d_1 h}}, \tag{31}$$

$$a_F = a_1 \frac{4}{3 \left(1 + \frac{k_1}{d_1 h} \right) + \left(1 + \frac{k_1}{2d_1 h} \right)^{-1}}. \tag{32}$$

Here k_i, a_i, d_i are the thermal conductivity, thermal diffusivity and thickness of layer respectively.

The same parameters obtained by the method of open cell are the following:

$$k_R = k_1, \quad \text{and} \quad a_R = a_1 \frac{1}{1 + \frac{k_1}{2d_1 h}}. \tag{33}$$

It is evidently that as the thermal conductivity so the thermal diffusivity depend essentially on the degree of the thermal transparence of the interface contact. The measurements by closed and open cells leads to the same results only in the case of isothermal contact, but it is already the one-layer sample.

3. Let us assume that the condition (23) is fulfilled and the additional relation $q_1 d_1 \ll q_2 d_2$ takes place. The direct calculations show that

$$k_F = k_2 \frac{d}{d_2} \left(1 + \frac{k_2}{d_2 h} \right)^{-1}, \tag{34}$$

$$a_F = a_2 \left(\frac{d}{d_2} \right)^2 \left(1 + \frac{k_2}{d_2 h} \right), \tag{35}$$

and

$$k_R = k_2, \quad a_R = a_2 \left(\frac{d}{d_2} \right)^2. \tag{36}$$

In this case the effective parameters depends only on the second layer heat parameters, the interface thermal conductivity and the geometric sizes of the layers. The effective thermal diffusivities measured by the different methods coincides when the interface thermal contact is isothermal ($a_F = a_R$). At the same time the effective thermal

conductivities differ by the factor determining the ratio of the sample thickness and the second layer thickness. The criterion of the ideal heat transparency of the interface is equal for the both types of measurements and is determined by the Eq.(30).

4. If the contrary relation $q_2d_2 \ll q_1d_1$ takes place then

$$k_F = k_1 \frac{d}{d_1} \left(1 + \frac{k_1}{d_1 h} \right)^{-1}, \quad (37)$$

$$a_F = a_1 \left(\frac{d}{d_1} \right)^2 \frac{1}{3 \frac{k_1}{d_1 h} + \left(1 + \frac{k_1}{d_1 h} \right)^{-1}}, \quad (38)$$

and

$$k_R = k_2, \quad a_R = a_1 \left(\frac{d}{d_1} \right)^2 \left(1 + \frac{2k_1}{d_1 h} \right). \quad (39)$$

We see that the effective parameters depend only on the first layer heat parameters, the interface thermal conductivity and the layer thickness. The condition of isothermality of the interface contact for all effective thermal parameters is determined by the inequality

$$h \gg \frac{k_1}{d_1}. \quad (40)$$

The effective values of the thermal conductivity and the thermal diffusivity are defined by the thermal and geometric parameters of the most thick layer.

II. Thick layers

Let us consider both layers are thick ($q_1d_1 \gg 1$, $q_2d_2 \gg 1$) and the correlation between its thicknesses is arbitrary.

For FSIM we obtain:

$$k_F = k_1, \quad (41)$$

$$a_F = a_1. \quad (42)$$

The influence of the interface and the second layer is very negligible under measurements of the thermal parameters from the side $x=0$, and an instrument records the first layer parameters only. The fact is that a thermal wave damping length is the same as a thermal diffusion length^[18], therefore a thermal wave damps in the first layer

The another situation takes place when RSIM is applied. One can obtain from the Eq. (18) that:

$$k_R = \frac{k_2 \cos(q_1d_1 + q_2d_2) \exp(q_1d_1 + q_2d_2 - qd)}{2 \cos(qd)} \times \left\{ 1 + \frac{k_1q_1}{k_2q_2} + \frac{k_1q_1}{h} [1 - \tan(q_1d_1 + q_2d_2)] \right\}, \quad (43)$$

$$a_R = \frac{wl^2}{2 \left[\arctan \left(\frac{\left[1 + \frac{k_1q_1}{k_2q_2} + \frac{k_1q_1}{2h} [1 + \tan(q_1d_1 + q_2d_2)] \right]}{\left[\tan(q_1d_1 + q_2d_2) \right]} \right) \right]^2} \times \quad (44)$$

The equation (44) transforms to the well known Tominaga and Ito result^[8] if interface contact is isothermal ($h \gg \frac{k_1}{d_1}$):

$$\frac{d}{\sqrt{a_R}} = \frac{d_1}{\sqrt{a_1}} + \frac{d_2}{\sqrt{a_2}}. \quad (45)$$

The effective thermal conductivity in this case is equal to

$$k_R = \frac{k_2}{2} \left(1 + \frac{k_1}{k_2} \sqrt{\frac{a_2}{a_1}} \right). \quad (46)$$

The effective thermal conductivity becomes function not only of the thermal conductivities of each layer but it, also, depends on the layer's thermal diffusivities.

The correlation (45) is correct only within the model of the open cell experiment, ideal heat interface and at the high modulation frequencies. Therefore it is the special case of the more general expression (44).

One can see that the effective thermal diffusivity is a complex function of all thermal parameters of both layers, its geometric sizes, the interface thermal conductivity and the chopper modulation frequency.

From the expressions (43) and (44) it is easy to get the effective parameters when the thermal thickness of layers are essentially different. In the situations when $q_1d_1 \gg q_2d_2$ or $q_1d_1 \ll q_2d_2$

$$k_R = \frac{k_2 \cos(q_i d_i) \exp(q_i d_i - qd)}{2 \cos(qd)} \times \left\{ 1 + \frac{k_1q_1}{k_2q_2} + \frac{k_1q_1}{h} [1 - \tan(q_i d_i)] \right\} \quad (47)$$

$$a_R = \frac{wl^2}{2 \left[\arctan \left(\frac{\left[1 + \frac{k_1q_1}{k_2q_2} + \frac{k_1q_1}{2h} [1 + \tan(q_i d_i)] \right]}{\left[\tan(q_i d_i) \right]} \right) \right]^2} \times \quad (48)$$

Here index i points to the most thick layer. The expressions (47) and (48) are simplified when ($h \gg \frac{k_1}{d_1}$). For the case when $q_1d_1 \gg q_2d_2$

$$k_R = \frac{k_2}{2} \left(1 + \frac{k_1}{k_2} \sqrt{\frac{a_2}{a_1}} \right), \quad (49)$$

$$a_R = \left(\frac{d}{d_1} \right)^2 a_1. \quad (50)$$

If $q_1 d_1 \ll q_2 d_2$,

$$\mathbf{k}_R = \frac{\mathbf{k}_2}{2} \left(1 + \frac{\mathbf{k}_1}{\mathbf{k}_2} \sqrt{\frac{\mathbf{a}_2}{\mathbf{a}_1}} \right), \quad (51)$$

$$\mathbf{a}_R = \left(\frac{d}{d_2} \right)^2 \mathbf{a}_2. \quad (52)$$

We can observe that effective thermal conductivity does not change in two last cases, while the effective thermal diffusivity is determined by thermal diffusivity of the most thermally thick layer and the ratio of the sample thickness to the thickness of this layer.

In conclusion let us note that different results received by FSIM and RSIM allows us to determine experimentally the value of interface thermal conductivity. For this purpose we can use for example the expressions (35), (36):

$$\mathbf{h}^{-1} = \frac{d_2}{\mathbf{k}_2} \left(\frac{\mathbf{a}_F}{\mathbf{a}_R} - 1 \right). \quad (53)$$

It is clear that this experimental method suggested may be applied when $\mathbf{a}_F \neq \mathbf{a}_R$.

Summary

Summarizing, we have devised a new approach to obtain the effective thermal conductivity and the effective thermal diffusivity of one-dimensional two-layer structures when the photoacoustic methods are used. The main idea of this approach is the simulation of the real two-layer structure by an effective one-layer sample, having the same geometric sizes and surface flux thermal boundary conditions. The equalities of temperatures on separate surfaces of this effective medium and the real two-layer system give possibility of calculation different effective thermal parameters.

One of our principal statements tells that under different photoacoustic measurements (closed cell or open cell) different values of thermal parameters exist in general case. Besides, the finite values of the interface thermal conductivity leads to essential modification of both the effective thermal conductivity and the effective thermal diffusivity.

ACKNOWLEDGEMENTS

This work has been partially supported by Consejo Nacional de Ciencia y Tecnología (CONACYT). Two of us (GGC) and (NMA) also wish to thank Centro de Investigación en Ciencia y Tecnología Avanzada (CICATA-I.P.N.) for helpful support.

References

- [1] M.S. Dresselhaus, T. Koga, X. Sun, S. B. Gronin, K. L. Wang and C. Chen. *Sixteenth International Conference on Thermoelectrics. Proceedings, ICT '97. August 26-29, 1997. Dresden, Germany*, p. 12, 1997.

- [2] C.D. Mahan and L.M. Woods, *Phys. Rev. Lett.* **80**, 4016 (1998).
- [3] S. M. Lee and David G. Cahill, *Appl. Phys. Lett.* **70** (22), 2957 (1997).
- [4] H. Vargas and L.C.M. Miranda, *Phys. Reports*, **161** (2), 43 (1988).
- [5] "Progress in Photothermal and Photoacoustic Science and Technology" edited by A. Mandelis (Prentice Hall, N.J., 1994).
- [6] M.J. Adams, G.F. Kirkbright, *Analyst*, **102** (1977) 281.
- [7] P. Charpentier, F. Lepoutre, L. Bertrand, *J. Appl. Phys.*, **53** (1) (1982) 608.
- [8] T. Tominaga and K. Ito, *Japan J. Appl. Phys.* **27**, 2 392 (1988).
- [9] A. Rosencwaig, A. Gersho, *J. Appl. Phys.*, **47** (1) 64 (1976).
- [10] Mansanares A. M., Bento A.C. Vargas H, Leite N. F. , M., *Phys. Rev.* **B42** 4477 (1995).
- [11] J.L. Lucio, J.J. Alvarado-Gil, O. Zelaya-Angel, H. Vargas, Miranda L. C. *Phys. Stat. Sol.*, (a) **150** 695 (1995).
- [12] González de la Cruz G. and. Gurevich Yu. G., *Phys. Rev.* **B51**, 2188 (1995).
- [13] Swarts, *Rev. Mod. Phys.* **61**, 605 (1989).
- [14] R. J. Stoner and M. J. Morris, *Phys. Rev.* **B48**, 16373 (1993).
- [15] S. M. Lee and David G. Cahill, *J. Appl. Phys.* **81** (6), 2590 (1997).
- [16] David G. Cahil and S. M. Lee, *J. Appl. Phys.* **83** (11), 5783 (1998).
- [17] F.G. Bass and Yu. G. Gurevich, *Soviet Physics Uspekhi*, **14**, No. 2, 113 (1971).
- [18] G. González de la Cruz and Yu. G. Gurevich, *Revista Mexicana de Física*, **45**(1), 46 (1999).
- [19] V. P. Babin, T. S. Gudkin, Z. M. Dashevsky, L. Dudkin, E. K. Iordanishvily, V. Y. Kaidonov, N. V. Kolomoets, O. M. Narva, L.S. Stilbans, *Sov. Phys. Semicond.* , **8** (4), 478 (1974).
- [20] Yu. G. Gurevich, G. N. Logvinov, *Journal of Thermoelectricity*, No2, p.8-13 (1998).